Phan I : VAT LY SONG

I-PHÖÔNG TRÌNH SONG ÑIEN TÖ:

1.Cac phöông trình Maxwell:

Tröông ñien tö trong chan khong vao thôi ñiem t nao ño ñöôc xac ñònh bôi vectô cöông ño ñien tröông $\vec{E}(\vec{r},t)$ va vectô cam öng tö $\vec{B}(\vec{r},t)$ vôi $\vec{r}$ la vectô vò trí tai ñiem ñang xet

Löc tac dung len ñien tích thö Q chuyen ñong vôi van toc $\vec{v}$ ñöôc bieu dien thong qua $\vec{E}$ va $\vec{B}$ nhö sau:

$$\vec{F} = Q\vec{E} + Q(\vec{v} \wedge \vec{B})$$

Nguon cua tröông ñien tö la cac ñien tích va dong ñien ,ñe ñac tröng cho cac ñai löông ño ngöôi ta dung mat ño ñien tích va vectô mat ño dong ñien $\vec{j}$. Cac phöông trình Maxwell bieu dien moi lien he giöa sö bien thien cua tröông ñien tö $(\vec{E},\vec{B})$ vôi cac nguon cua no (ñien tích ,dong ñien)

- Pt M-Φ : div$\vec{B}$ = 0 bao toan tö thong
- Pt M-F : $rot\vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$ cam öng ñien tö
- Pt M-G : div$\vec{E}$ = $\dfrac{\rho}{\varepsilon_0}$
- Pt M-A : $rot\vec{B} = \mu_0 \vec{j} + \mu_0 \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t}$

$\vec{j}$: dong ñien dan

$\varepsilon_0 \dfrac{\partial \vec{E}}{\partial t}$: dong ñien dòch

$\varepsilon_0 = \dfrac{1}{36\pi}.10^{-9}$ (F.m^{-1}) :hang so ñien

$\mu_0 = 4\pi .10^{-7}$ H.m^{-1}

2.Cac phöông trình lan truyen song:

$$rot(rot\vec{E}) = rot(-\frac{\partial \vec{B}}{\partial t}) = -\frac{\partial}{\partial t}(rot\vec{B})$$

Mat khac

$$rot(rot\vec{A}) = grad(div\vec{A}) - \Delta\vec{A}$$

$$M - A \Rightarrow -\frac{\partial}{\partial t}(rot\vec{B}) = -\mu_0\frac{\partial\vec{j}}{\partial t} - \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\Rightarrow \Delta\vec{E} - \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2} = grad(\frac{\rho}{\varepsilon_0}) + \mu_0\frac{\partial\vec{j}}{\partial t}(a)$$

$$M - A \Rightarrow rot(rot\vec{B}) = \mu_0 rot\vec{j} + \mu_0\varepsilon_0 rot(\frac{\partial\vec{E}}{\partial t})$$

$$= \mu_0 rot\vec{j} + \mu_0\varepsilon_0\frac{\partial}{\partial t}(rot\vec{E})$$

$$M - F \Rightarrow \mu_0\varepsilon_0\frac{\partial}{\partial t}(rot\vec{E}) = -\mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2}$$

$$\Rightarrow grad(div\vec{B}) - \Delta\vec{B} = \mu_0 rot\vec{j} - \mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2}$$

$$\Leftrightarrow \Delta\vec{B} - \mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2} = \mu_0 rot\vec{j}(b)$$

(a) va (b) la phöông trình lan truyen cua tröông

3.<u>Tröông hôp khong co nguon</u>:($\rho = 0, \vec{j} = 0$)

Cac pt lan truyen cua ñien tröông E va tö tröông B luc ño co dang cua pt D'Alembert:

$$\Delta\vec{E} - \frac{1}{C^2}\frac{\partial^2\vec{E}}{\partial t^2} = 0 \quad ; \quad \Delta\vec{B} - 1/C^2\frac{\partial^2\vec{B}}{\partial t^2} = 0 \quad \text{(c)}$$

Vôi $C^2 = 1/\varepsilon_0\mu_0$:van toc truyen trong chan khong.

- Toan tö D'Alemert: $\Box = \Delta - 1/C^2\frac{\partial^2}{\partial t}$
- ⇨ $\Box\vec{E} = 0$; $\Box\vec{B} = 0$
- ⇨ Ñoi vôi moat thanh phan cua tröông (a),co the bieu dien döôi dang $\Box a = 0$

4.<u>Cac the cua tröông:</u>

Div(rot $\vec{A}$) = 0

$M - \Phi : div\vec{B} = 0$ =>ton tai moat tröông vectô $\vec{A}$: $\Rightarrow \vec{B} = rot(\vec{A})$

$$M - F \Rightarrow rot\vec{E} = -\frac{\partial\vec{B}}{\partial t} = -\frac{\partial}{\partial t}(rot\vec{A}) = -rot(\frac{\partial\vec{A}}{\partial t})$$

$$\Rightarrow rot(\vec{E} + \frac{\partial\vec{A}}{\partial t}) = 0$$

=>tröông xoay ton tai tröông vo höông V :

$$\vec{E}+\frac{\partial\vec{A}}{\partial t}=-gradV$$
$$(rot(gradV)=0)$$

Tom lai ,tröông ñien tö ($\vec{E}$, $\vec{B}$) co moat cap the ($\vec{A}$,V) lien he vôi chung qua bieu thöc :

$$\vec{E} = -\frac{\partial\vec{A}}{\partial t}-gradV \quad ; \quad \vec{B} = \text{rot}\,\vec{A}$$

Neu $\vec{A}$ la vectô the cua tröông ñien tö thì :

$\vec{A}$' = $\vec{A}$ + gradf cung la vectô the

V la the cua tröông thì

V' = V - $\frac{\partial f}{\partial t}$ cung la the.

Trong so nhöng cap the cua mot tröông ñien tö xac ñònh ton tai moat cap the thoa ñieu kien chuan Lorentz :

$$div\vec{A}+\mu_0\varepsilon_0\frac{\partial V}{\partial t}=0$$

$$\frac{\rho}{\varepsilon_0}=div\vec{E}=-div(gradV+\frac{\partial A}{\partial t})=-\Delta V-\frac{\partial}{\partial t}(div\vec{A})$$

$$\Rightarrow \Delta V-\varepsilon_0\mu_0\frac{\partial^2 V}{\partial t^2}=-\frac{\rho}{\varepsilon_0}$$

$$rot\vec{B}=rot(rot\vec{A})=\mu_0(\vec{j}+\varepsilon_0\frac{\partial\vec{E}}{\partial t})=\mu_0\vec{j}+\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$=\mu_0\vec{j}-\mu_0\varepsilon_0 grad(\frac{\partial V}{\partial t})-\varepsilon_0\mu_0\frac{\partial^2 A}{\partial t^2}$$

$$grad(div\vec{A})-\Delta\vec{A}=\mu_0\vec{j}+grad(div\vec{A})-\varepsilon_0\mu_0\frac{\partial^2 A}{\partial t^2}$$

$$\Rightarrow \Delta\vec{A}-\varepsilon_0\mu_0\frac{\partial^2 A}{\partial t^2}=-\mu_0\vec{j}$$

II- SONG ÑIEN TÖ PHANG CHAY ÑIEU HOA LIEN TIEP:(OPPH)

1.Mô ñau:

- Mat song : la tap hôp cac vò trí ma ño lôn cua tröông khong ñoi vao thôi ñiem xac ñònh.
- Song phang (OP) la song co mat song la mot ho cac mp vuong goc vôi phöông truyen song xac ñònh $\vec{u}(|\vec{u}|=1)$
- Song phang lien tiep (OPP) la song phang truyen theo phöông va chieu xac ñònh ,ham song co dang:

a(M,t) = $f(\vec{u}.\vec{r}-ct)$

Nghiem cua phöông trình D'Alembert la toi hôp cac song phang lien tiep theo mot phöông $\vec{u}$ nao ñoi .

- Song phang ñieu hoa lien tiep :la song phang lien tiep ma ham song co dang sin hoac cos.

$$a(\vec{r},t) = A\cos(\omega t - \vec{k}\vec{r} - \Phi)$$

So song k= $\omega / C = 2\pi / \lambda$

Vectô song $\vec{k}$ = k$\vec{u}$.

- Song ñien tö phang ñieu hoa lien tiep la nghiem cua phöông trình Maxwell ma 6 thanh phan cua tröông ñien tö co cung tan so goc ω va cung vectô song $\vec{k}$

Co the bieu dien tröông ñien tö döôi dang phöc :

$$\underline{\vec{E}} = \underline{\vec{E}_0} e^{j(\omega t - \vec{k}\vec{r})} \qquad \underline{\vec{B}} = \underline{\vec{B}_0} e^{j(\omega t - \vec{k}\vec{r})}$$

Cac toan tö ñao ham tac dung len tröông phöc töông ñöông vôi phep nhan :

$$\frac{\partial}{\partial t} = j\omega \qquad ; \qquad \vec{\nabla} = j\vec{k}$$

2.<u>Cau truc cua OPPH trong chan khong</u>:

Bieu dien pt Maxwell bang cach sö dung toan tö rabla $\vec{\nabla}$

$$\vec{\nabla}\vec{E} = 0 \qquad \vec{\nabla} \wedge \vec{E} = -\partial \vec{B} / \partial T$$

$$\vec{\nabla}\vec{B} = 0 \qquad \vec{\nabla} \wedge \vec{B} = \varepsilon_0 \mu_0 \partial \vec{E} / \partial t$$

=>Döôi dang phöc :

$$-j\vec{k}.\vec{E} = 0(1) \qquad -j\vec{k} \wedge \vec{E} = -j\omega \underline{\vec{B}}(3)$$

$$-j\vec{k}.\vec{B} = 0(2) \qquad -j\vec{k} \wedge B = \varepsilon_0 \mu_0 j\omega \underline{\vec{E}(4)}$$

(1) => $\vec{k}.\underline{\vec{E}} = 0 \Rightarrow \vec{u}\underline{\vec{E}} = 0$

$$\to \mathrm{Re}(\vec{u}.\underline{\vec{E}}) = o \Rightarrow \vec{u}\,\mathrm{Re}(\underline{\vec{E}}) = 0 \Rightarrow \vec{u}\vec{E} = 0$$

Mot ach töông tö => $\vec{u}\vec{B} = 0$.

→ Song ñien tö phang ñeu hoa lien tiep trong chan khong la song ngang.

(3) => $\underline{\vec{B}} = \vec{k} \wedge \vec{E} = k.\vec{u} \wedge \underline{\vec{E}}(3')$ $\qquad (4) \Rightarrow k\vec{u} \wedge \underline{\hat{B}} = -\frac{\omega}{C^2}\underline{\vec{E}}(4')$

The (3') vao (4') :

$$k.\vec{u} \wedge \left(\frac{k}{\omega}\vec{u} \wedge \underline{\vec{E}}\right) = -\frac{\omega}{C^2}\underline{\vec{E}}$$

Ta co: $.\vec{u} \wedge (\vec{u} \wedge \underline{\vec{E}}) = (\vec{u}.\underline{\vec{E}}).\vec{u} - (\vec{u}.\vec{u}).\underline{\vec{E}}) = -\underline{\vec{E}}$

=> $k^2 = \frac{\omega^2}{c^2}$

(3') => $\vec{B} = \frac{k}{\omega}\vec{u} \wedge \vec{E} = \frac{\vec{u} \wedge \vec{E}}{c}$

Lay phan thöïc :

$$\vec{B} = \mathrm{Re}(\frac{\vec{u} \wedge \vec{E}}{c}) = \frac{\vec{u} \wedge \vec{E}}{c}$$

=> $(\vec{u}, \vec{E}, \vec{B})$ tao thanh mot tam dien thuan

Mat khac ty soi giöia tröông ñien va töi la:

$$\frac{E(M,t)}{B(M,t)} = c$$

Ñien tröông va töi tröông cua OPPH ñong pha .Cac tính chat tren cung ñung vôi OPP.

3.Söi phan cöïc cua OPPH:

Trong OPPH phöông cua ñien tröông $\vec{E}$ trong mat phang vuong goc vôi phöông truyen song $\vec{u}$ chöùa ñöôïc xac ñònh .Phöông cua vectô $\vec{E}$ ñöôïc goi la phöông phan cöïc cua song

Xet trong he toai ño Descartes ,gia söi song truyen theo phöông z

$$\vec{E} = \begin{cases} E_{0x}\cos(\omega t - kz + \phi_x) \\ E_{0y}\cos(\omega t - kz + \phi_y) \\ 0 \end{cases}$$

Mot khi biet ñöôïc ñien tröông $\vec{E}$ ta coi thei xac ñònh ñöôïc töi tröông bôi cau truc OPPH.

Tai mot vò trí z = z_0 coi ñònh ,ta coi thei viet söi bien thien cua ñien tröông nhö sau :

$E_x = E_{0x}\cos(\omega t)$

$E_y = E_{0y}\cos(\omega t - \varphi)$

Vôi $\varphi = \varphi_x - \varphi_y$: ño tre pha cua E_y ñoi vôi E_x

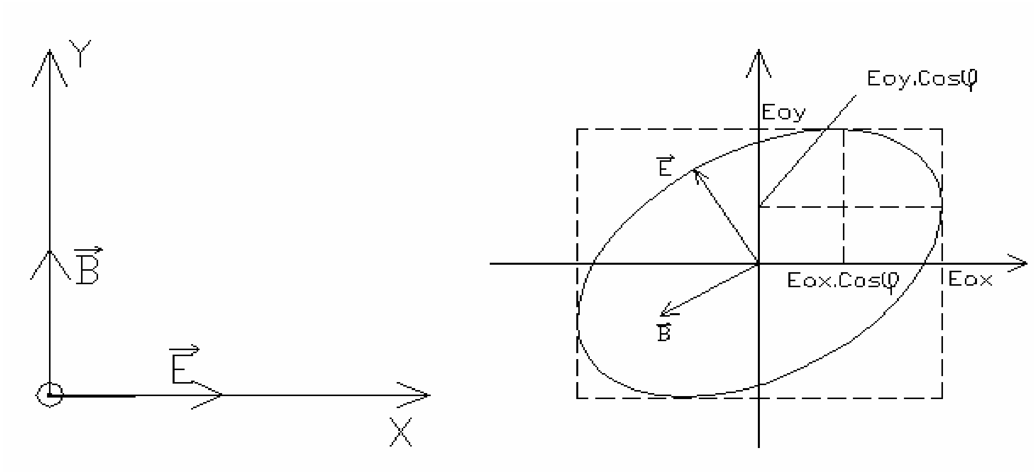
Ñau mut cua vectô ñien tröông dòch chuyen trong mp (xOy) ,hình chöi nhat coi canh $2E_{0x}$ va $2E_{0y}$ tren ñöông ellipse coi pt:

$$(\frac{E_x}{E_{0x}})^2 + (\frac{E_y}{E_{0y}})^2 - 2(\frac{E_x}{E_{0x}})(\frac{E_y}{E_{0y}})\cos\varphi = \sin^2\varphi$$

Ñei xac ñònh chieu chuyen ñong doc theo ellipse ,ta xet vao thôi ñiem t=0 ,khi ñoi $E_x = E_{0x}$ va :

$$(\frac{dE_y}{dt})_{t=0} = E_{0y}\omega.\sin\varphi$$

⇨ chieu quay ñöôïc chæ ra bôi dau cua $\sin\varphi$.



- Neáu chieàu quay thuaän chieàu kim ñoàng hoà :soùng phaân cöïc ellipse traùi, sinφ > 0
- Neáu φ = 0 hoaëc φ = $\pm\pi$,ñaàu muùt cuûa E dòch chuyeån treân ñöôøng thaúng xaùc ñònh ,ta coù phaân cöïc thaúng

Noùi chung moät soùng phaân cöïc ellipse coù theå xem laø toång cuûa 2 soùng phaân cöïc thaúng theo hai phöông vuoâng goùc vôùi nhau => moïi soùng ñieän töø trong chaân khoâng laø söï toång hôïp cuûa caùc soùng phaúng ñieàu hoaø lieân tieáp phaân cöïc thaúng.

- Neáu φ = $\pm\pi$ /2 vaø $E_{0x} = E_{0y}$ ta coù phaân cöïc troøn.

4.<u>Söï truyeàn naêng löôïng cuûa OPPH</u>:

Maät ñoä naêng löôïng cuûa tröôøng ñieän töø:

$$e = \frac{\varepsilon_0 E^2}{2} + \frac{B^2}{2.\mu_0}$$

Ñoái vôùi OPPH : B = E/c => e = $\varepsilon_0 E^2$ = $\dfrac{B^2}{\mu_0}$

⇨ naêng löôïng ñöôïc phaân boá ñeàu döôùi daïng ñieän vaø töø.

Ñoái vôùi moät soùng OPPH truyeàn theo phöông cuûa truïc Ox ,tröôøng ñieän töø coù daïng:

$$\underline{\vec{E}} = \underline{\vec{E}_0} e^{j(\omega t - kx)}$$

$$\underline{\vec{B}} = \frac{\vec{u}_x \wedge \vec{E}_0}{c} e^{j(\omega t - kx)}$$

Giaù trò trung bình cuûa e:

⇨ <e> = <$\varepsilon_0 E^2$> = $\varepsilon_0 \dfrac{1}{2}\mathrm{Re}(\underline{\vec{E}}.\underline{\vec{E}}^*)$ = $\dfrac{\varepsilon_0}{2}\left|\vec{E}_0\right|^2$

*<u>Vectô Poynting:</u>

Coing suait cuia soing ñiein töi (W) ñi qua moit diein tích S baing doing cuia vectô poymting $\pi = \dfrac{\vec{E} \wedge \vec{B}}{\mu_0}$ (vectô doing naing löôing W_m ñi qua dieim tích ño =

$$\Phi = \int_S \vec{\pi}.d\vec{S} \quad)$$

Ñoii vôii OPP $\vec{\Pi} = \dfrac{\vec{E} \wedge \vec{B}}{\mu_0} = \dfrac{\vec{E} \wedge (\vec{u} \wedge \vec{E})}{\mu_0 c} = c\varepsilon_0 E^2 \vec{u}.$

Ñoii vôii soing OPPH coi tain soi ω , giai trò trung bình $<\Phi>$ cuia coing suait truyein qua mait S vuoing goic vôii phöông truyein $\vec{u}$

$$<\Phi> = <\vec{\Pi}>.\vec{S} = 1/2\text{Re}\left(\frac{\underline{\vec{E}} \wedge \underline{\vec{B}}^*}{\mu_0}\right).\vec{S} = \frac{1}{2}c\varepsilon_0 \left|\underline{E}_0\right|^2.S$$

Ta coi: $<\vec{E}(t) \wedge \vec{B}(t)> = \dfrac{1}{2}\text{Re}(\underline{\vec{E}}(t) \wedge \underline{\vec{B}}^*(t)) = \dfrac{1}{2}\text{Re}(\underline{\vec{E}}_m(t) \wedge \underline{\vec{B}}^*{}_m(t))$

$$= \frac{1}{2}\text{Re}(\vec{E}_m \wedge \vec{B}_m e^{j(\varphi_1 - \varphi_2)}) = \frac{1}{2}\vec{E}_m \wedge \vec{B}_m \cos(\varphi_1 - \varphi_2)$$

*Vain toic truyein naing löôing:

V_e : vain toic truyein naing löôing.

$<\Pi>.S.\delta t$: naing löôing truyein qua diein tích S vuoing goic vôii phöông truyein soing trong khoaing thôii gian δt

$S.v_e.\delta t < e >$: naing löôing chöia trong thei tích $S.v_e.\delta t$

$$S.v_e.\delta t < e > = < \Pi > .S.\delta t$$

$$=> \quad v_e = \frac{<\vec{\Pi}>}{<e>}$$

Ñoii vôii OPPH : v_e = c.

$$\vec{E} = E_{0y}\cos(\omega t - kx)\vec{u}_y \pm E_{0z}\sin(\omega t - kx)\vec{u}_z$$

$$=> \quad \vec{\Pi} = \frac{<\vec{E}^2>}{\mu_0 c}\vec{u} = \frac{\vec{u}}{\mu_0 c} < E_{0y}^2 \cos^2(\omega t - kx) + E_{0z}^2 \sin(\omega t - kx) >$$

$$= \left(\frac{E_{oy}^2 + E_{0z}^2}{2\mu_0 c}\right).\vec{u}$$

$$< e > = \varepsilon_0 < E^2 > = \varepsilon_0 < E_{0y}^2 \cos^2(\omega t - kx) + E_{0z}^2 \sin(\omega t - kx) >$$

$$= \varepsilon_0 \frac{E_{0y}^2 + E_{0z}^2}{2}$$

Vecto Poynting phöic : $\vec{\Pi} = \dfrac{\underline{\vec{E}} \wedge \underline{\vec{B}}^*}{\mu_0}$

$$\underline{\vec{E}} = \underline{\vec{E}}_0.\exp\left[j\left(\omega t - \vec{k}\vec{r}\right)\right]$$

Ñoii vôii OPPH :

$$\underline{\vec{B}} = \frac{\vec{u} \wedge \underline{\vec{E}}_0}{c}.\exp\left[j\left(\omega t - \vec{k}\vec{r}\right)\right]$$

$$\underline{\vec{B}}^* = \frac{\vec{u} \wedge \underline{\vec{E}}_0^*}{c}.\exp\left[-j\left(\omega t - \vec{k}\vec{r}\right)\right]$$

=> $\underline{\vec{\Pi}} = \dfrac{\underline{\vec{E}} \wedge \underline{\vec{B}}^*}{\mu_0} = \underline{\vec{E}}_0 \wedge \left(\dfrac{\vec{u} \wedge \underline{\vec{E}}_0^*}{\mu_0 c}\right) = \left(\dfrac{\underline{E}_0.\underline{E}_0^*}{\mu_0 c}\right).\vec{u} = \dfrac{E_0^2}{\mu_0 c}.\vec{u}$

=> $< d\phi > = < \vec{\Pi} > d\vec{S}$

OPPH: $< \vec{\Pi} > = \dfrac{< E^2 >}{\mu_0 c}\vec{u} = \dfrac{c\varepsilon_0 E_0^2}{2}\vec{u}$